CS 1200 Discrete Math

Math Preliminaries

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- Course Objective:
  - Mathematical way of thinking in order to solve problems

Math Preliminaries

- Variable: A variable is simply a place holder.
- One of the most important use of variables is that it gives one the ability to refer to quantities unambiguously through out a lengthy mathematical discussion while not restricting one to consider only specific values for that.

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- Examples
  - Is there a number such that doubling it and adding 3 be equal to its square?

$$2x + 3 = x^2 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x + 1)(x - 3) = 0 \Rightarrow$$
$$(x + 1) = 0 \text{ and } (x - 3) = 0 \Rightarrow$$
$$x = -1 \text{ and } x = 3$$

  - Are there numbers such that sum of their square is equal to the square of their sum?

$$a^2 + b^2 = (a + b)^2 = a^2 + b^2 + 2ab \Rightarrow 2ab = 0 \Rightarrow ab = 0 \Rightarrow$$
$$a = 0 \text{ or } b = 0$$

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- **Universal Statement:** A universal statement expresses a certain property that is true for all elements in a set.
- Note, universal statements contain some variation of the word "for all".

Example: All dogs are mammal

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- Conditional Statement: A Conditional statement expresses that if one thing is true then some other thing also is true.
- Note, conditional statements contain some variation of the word "if-then".

Example: If an animal is a dog, then *animal* is a mammal

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- **Universal Conditional Statement:** A universal conditional statement is a statement that is both universal and conditional.

Example: For all animals *a*, if *a* is a dog, then *a* is a mammal

- Note, universal conditional statements can be written in different ways as pure universal or pure conditional statements.

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- For example the statement

  For all animals *a*, if *a* is a dog, then *a* is a mammal

Can be written as:

Pure conditional

If *a* is a dog, then *a* is a mammal, or
If an animal is a dog, then *animal* is a mammal

Pure universal

All dogs are mammals, or
For all dogs *a*, *a* is a mammal

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- Existential Statement: Existential statement is a statement that there is at least one thing for which the property is true.

Example: There exist a prime number that is even

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- **Universal Existential Statement:** is a statement that is universal since its first part is universal (i.e., certain property is true for all objects of a given type) and it is existential since its second part asserts the existence of something.

Example: Every real number has an additive inverse

Implies universality

Asserts existence of something

- Existential Universal Statement: is a statement that is existential since its first part asserts that a certain object exist and it is universal since its second part says that the object satisfies property for all things of a certain kind.

Example: There is a positive integer that is less than or equal to every positive integer

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- Note, the statement

There is a positive integer that is less than or equal to every positive integer

can be rewritten in several ways:

There is a positive integer m that is less than or equal to every positive integer, or
There is a positive integer m that every positive integer is greater than or equal to m, or
There is a positive integer m with the property that all positive integer n, $m \leq n$, or
$\exists\, m > 0$, *such that* $\forall\, n > 0, m \leq n$

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- **Set**: A set is a collection of objects (elements) with some common characteristics.

  Example: set of all integers

- A set is represented by a capital letter: set S.

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- Set membership: if S is a set then the notation $x \in S$ represents x as a member of S.
- Similarly the notation $x \notin S$ means that x is not a member of S.
- A set can be represented using self-roster notation (i.e., by writing all of its elements between braces:

Example: {1, 2, 3} is a set whose members are 1, 2, and 3.

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- Note, in a set the order of elements is immaterial. In addition, duplication of elements does not change a set (axiom of extension).

- Note, an element of a set could be a set itself.

Example: $\{1, \{2\}\}$

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- Some common sets:

  R set of real numbers

  Z set of integers

  Q set of rational numbers

  N set of natural numbers

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- Some conventions:

  R^+ denotes set of positive real numbers

  R^- denotes set of negative real numbers

  Z^{nonneg} denotes set of non-negative integers

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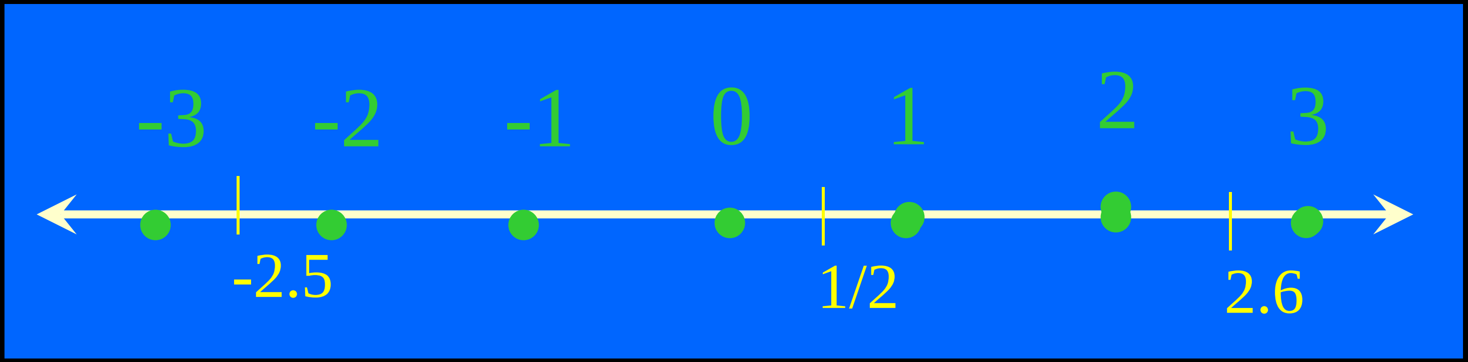
- Real number line



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- Real number line shows a continuity (no holes). This is in contrast to the notion of discrete mathematics.

Example: Set of integers are

-3 -2 -1 0 1 2 3

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- Formal definition of a set: Let *S* be a set and let *P(x)* be a property that elements of *S* satisfy (i.e., a predicate that is true), then *S* is defined as:

$$S = \{x \in S \mid P(x)\} \quad \text{or in short } \{x \mid P(x)\}$$

- Cardinality of *S* denoted as |S| is the number of elements of *S*.

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- Subset: Assume *A* and *B* are two set, then *A* is called a subset of *B* if every element of *A* is an element of *B*.

$A \subseteq B \Rightarrow$ for all x such that $x \in A \Rightarrow x \in B$

A is contained in *B* or *B* contains *A*.

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- Similarly A $\nsubseteq$ B indicates that *A* is not a subset of *B*.

$$A \nsubseteq B \Rightarrow \text{there exist } x \text{ such that } x \in A \text{ and } x \notin B$$

- Proper subset: *A* is proper subset of *B*, if every element of *A* is an element of *B* and there exist an element of *B* which is not an element of *A*

$$A \subset B \Rightarrow (\text{every } x \in A \Rightarrow x \in B) \text{ and } \exists\, (y \in B \wedge y \notin A)$$

- **Ordered Pair:** Given elements *a* and *b*, the symbol (a, b) shows that *a* is the first element and *b* is the second one. Hence

$$(a, b) \neq (b, a)$$

- Note

$$(a, b) = (c,d) \text{ iff } a = c \text{ and } b = d$$

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- Set Operators:
  - Cartesian Product: Given two sets *A* and *B*, the Cartesian product of *A* and *B* denoted as A X B is a set of all ordered pair (a, b) such that *a* is an element of *A* and *b* is an element of *B*.

$$A \times B \Rightarrow \{ (a, b) \mid a \in A \wedge b \in B \}$$

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- Set Operators:
  - Set Union: Given two sets *A* and *B*, the set union of *A* and *B* denoted as $A \cup B$ is:

$$A \cup B \Rightarrow \{\, x \mid x \in A \text{ or } x \in B \,\}$$

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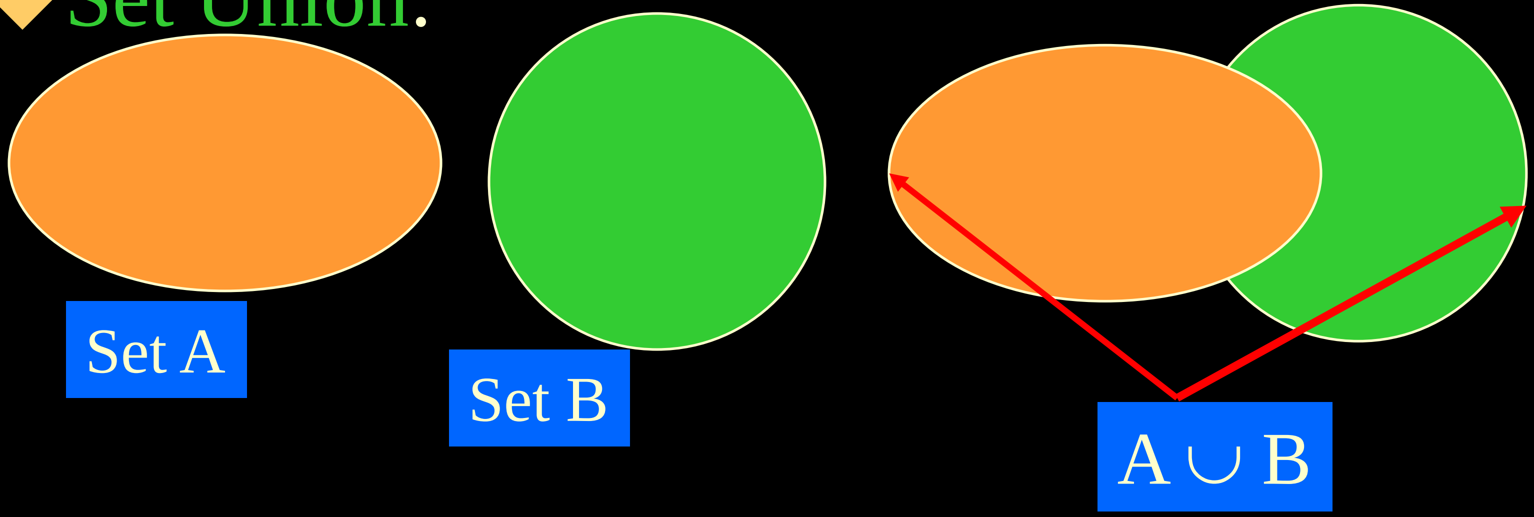
- Set Union:



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- Set Operators:
  - Set intersection: Given two sets *A* and *B*, set intersection of *A* and *B* denoted as $A \cap B$ is:

$$A \cap B \Rightarrow \{ x \mid x \in A \text{ and } x \in B\}$$

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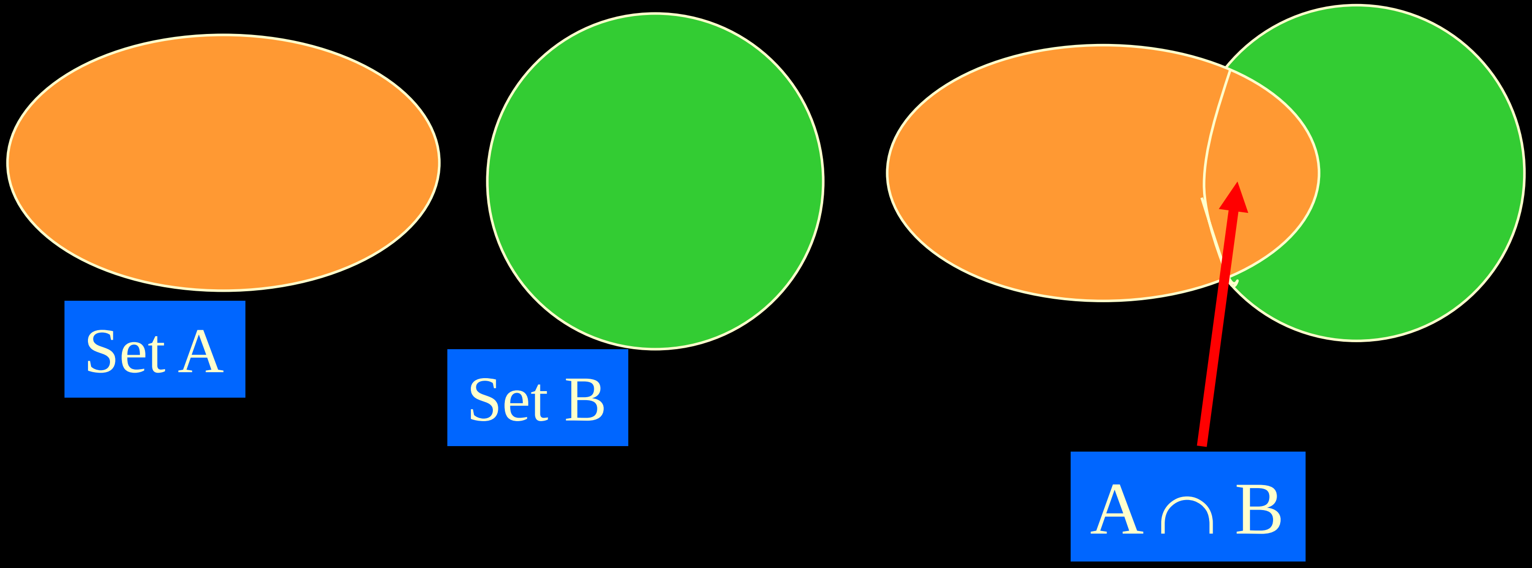
- Set intersection:



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- Set Operators:
  - Set difference: Given two sets *A* and *B*, the set difference of *A* and *B* denoted as A - B is:

$$A - B \Rightarrow \{ x \mid x \in A \text{ and } x \notin B \}$$

  - Note $A – B \neq B – A$

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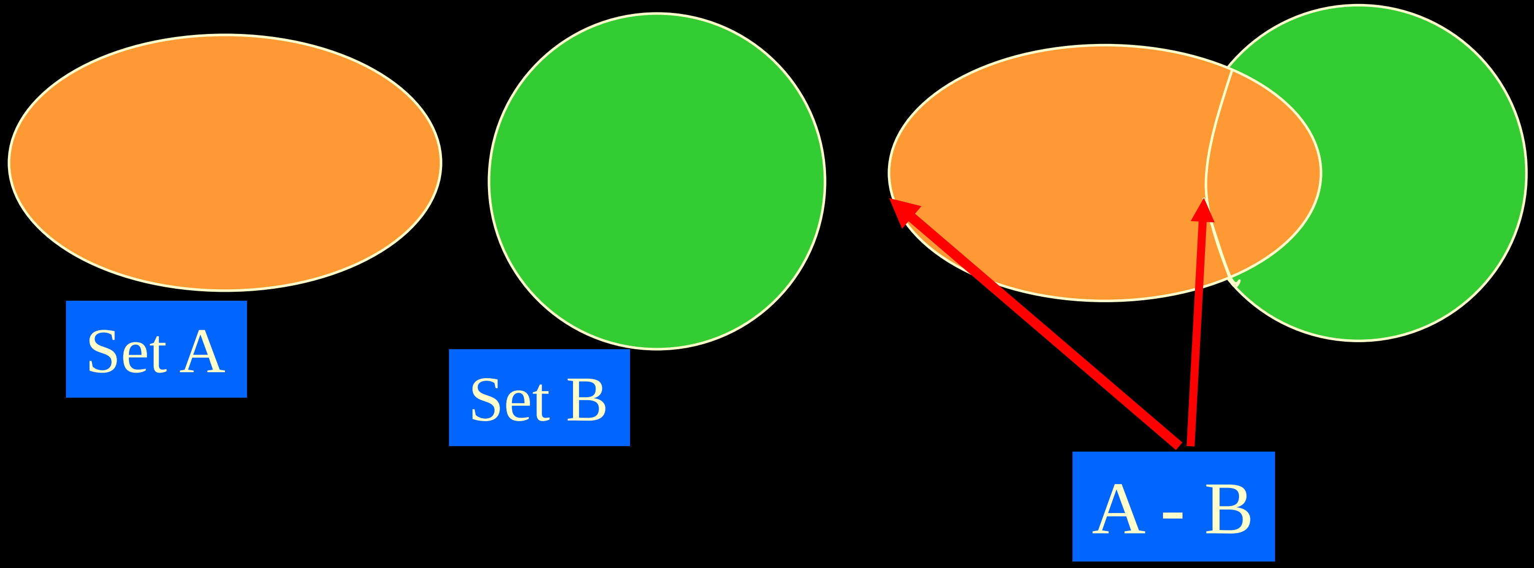
- Set Difference:



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- Some observations:

$$\bar{A} = \{ x \mid x \in U \text{ and } x \notin A \}$$

$$A \cup \emptyset = A$$

$$A \cap \emptyset = \emptyset$$

$$A - \emptyset = A$$

$$\overline{\emptyset} = U$$

$$\bar{\bar{A}} = A$$

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- Power set: Assume S is a set, then the power set of S has $2^{|S|}$ elements:

Example: Assume S is {a, b, c}
Then the power set of S denoted as P(S) is:
P(S) = {Ø, a, b, c, ab, ac, bc, abc}

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- Relations: Let *A* and *B* be two sets. We define a relation *R* from *A* to *B* as a subset of AXB such that any pair $(x, y) \in A \times B$ satisfy *R* (i.e., *x* is related to *y* by *R* denoted as *x R y*). A is called the domain and B is called the co-domain.
- Symbolically:

$$x\ R\ y \Rightarrow (x, y) \in R$$
$$x\ \not{R}\ y \Rightarrow (x, y) \notin R$$

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- Assume A = {0, 1, 2} and B = {1, 2, 3} and let us define the following relationship between elements of *A* and *B*: $x \in A$ is related to $y \in B$ if x<y, then we have:

A X B $\Rightarrow$ {(0, 1), (0, 2), (0, 3), (1, 1), (1, 2), (1, 3)
(2, 1), (2, 2), (2, 3)}

Ordered element of A X B related based on < are:
{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)}

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- A relation can be graphically represented as an arrow diagram in which:
  1) each set is represented as a region with its elements as a point, and
  2) there is a directed edge from each element of the domain to its corresponding element of co-domain that satisfy the relation.

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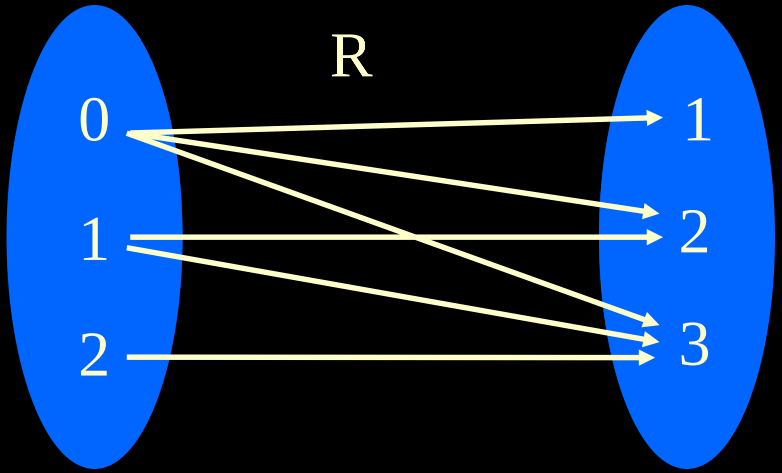
- Assume A = {0, 1, 2} and B = {1, 2, 3} and let us define the following relationship between elements of *A* and *B*: $x \in A$ is related to $y \in B$ if x<y, then arrow diagram representation of R is:



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- Function: A function F from a set A to a set B is a relation that satisfies the following two properties:

  1: for each element $x \in A$ $\exists$ $y \in B$ such that $(x, y) \in F$, and

  2: for all elements $x \in A$ and $y, z \in B$,

  if $(x, y) \in F$ and $(x, z) \in F$ then $y = z$.